ENEE 610
Tu 09/29/20

Note Title 9/19/2020

Homework 4
Circulator Y matrix

Duality. Companion matrix state equations, State from semi-state; load from input via Richards, negative impedance converters, Laplace transform inversions & initial conditions. Simulink

For the 3-port circulator

$$S := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \qquad Id := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Y := (Id + S)^{-1} \cdot (Id - S) \qquad Y = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$Y^T = -Y$$

$$S = (1_n + Y)^{-1}(1_n - Y)$$

$$(1_n + Y)S = 1_n - Y$$

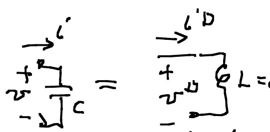
$$YS + Y = 1_n - S$$

$$Y(1_n + S)$$

$$Y = (1_n - S)(1_n + S)^{-1} = (1_n + S)^{-1}(1_n - S)$$

$$(1_n + S)(1_n - S) = 1_n + S - S - S^2 = 1_n - S^2 = (1_n - S)(1_n + S)$$

Dual: $\boxed{N, v, i} \xrightarrow[\text{dual}]{} \boxed{N^D,\ v^D = i,\ i^D = v}$



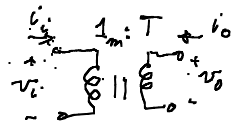
$v^D = L\, i^D$, $L = C$, $Y_{out} = \infty$ pole

$v^D = i = C \frac{dv}{dt} = C \frac{di^D}{dt}$

$v = Z i \Rightarrow i^D = Z v^D \Rightarrow Y^D = Z$

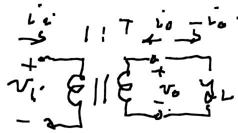
$2v^i = v + i$
$2v^n = v - i$ $\Rightarrow$ $2v^{i^D} = i^D + v^D = v + i$
$2v^{n^D} = v^D - i^D = i - v$

$$S^D = -S$$



$$v_o = T v_i \qquad i_i = -T^T i_o \quad \Big\} \quad P_{in} = v_o^T i_o + v_i^T i_i = v_i^T T^T i_o + v_i^T i_i = v_i^T [T^T i_o + i_i] = 0$$

for v_i can cancel v_i $\Rightarrow i_i = -T^T i_o$



$$-i_o = i_L = y_L v_o = y_L T v_i$$

$$i_i = -T^T i_o = -T^T(-y_L T) v_i \Rightarrow y_{in} = T^T y_L T \Rightarrow y_L = T^{-T} y_{in} T^{-1}$$

Semi-state Eqs: $E s X = A X + B u$, $y_{out} = C X$

y_{out} vs $u \Rightarrow$ transfer function matrix $= T_{ran}(s)$

$y_{out} = C X$, $(E s - A) X = B u \Rightarrow X = (E s - A)^{-1} B u$

$T_{ran}(s) = C (E s - A)^{-1} B$

Let $X = T x_n$

assume T^{-1} exists

$$E s T X_n = A T x_n + B u, \quad y_{out} = C T x_n$$

$$s T_1 (E T X_n) = T_1 A T x_n + T_1 B u \;;\; y_{out} = C T X_n$$

$$y_{out} = C T (\underbrace{s T_1 E T - T_1 A T}_{(T_1 (s E - A) T)^{-1}})^{-1} T_1 B u, \; C = T X_n$$

$$\underbrace{C T\, T^{-1} (s E - A)^{-1} T_1^{-1} \; T_1 B}_{C (s E - A)^{-1} B \cdot u}$$

if $T_1 = T^T$ then if the semi-state eqs. are like admittances it can be made by a transformer by choice of T_1 & T can diagonalize E

by singular value decomposition get $UEV = \mathrm{diag}\begin{pmatrix}\text{singular values} \\ \text{largest at top}\end{pmatrix}$

can bring E to $\begin{bmatrix} E_1 & & & \\ & E_2 & & \\ & & 0 & \\ & & & 0 \ddots \end{bmatrix} = \left[\begin{array}{c|c} E_0 & 0 \\ \hline 0 & 0 \end{array}\right]$ $\quad U = T_1, \; V = T$

E_0 non-singular

$$s\left[\begin{array}{c|c} E_0 & 0 \\ \hline 0 & 0 \end{array}\right]\begin{bmatrix} x_1 \\ \hline x_2 \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} u$$

to get state variable equations, eliminate x_2

$$0 = A_{21}x_1 + A_{22}x_2 + B_2 u; \quad x_2 = -A_{22}^{-1}(A_{21}x_1 + B_2 u)$$

$$s E_0 x_1 = A_{11}x_1 + A_{12}\left(-A_{22}^{-1}(A_{21}x_1)\right) + A_{12}(-B_2 u$$

$\therefore$ if A_{22}^{-1} exists then state-variable eqs. exist

$$s E_0 x_1 = [A_{11} - A_{12}A_{22}^{-1}A_{21}]x + [B_1 - A_{12}B_2]u$$

these are state variables, E_0 nonsingular, $\times E_0^{-1}$ gives

$$s 1_n x_1 = E_0^{-1}[A_{11} - A_{12}A_{22}^{-1}A_{21}]x + E_0^{-1}(B_1 - A_{12}B_2)u$$

$$= \tilde{A}x_1 + \tilde{B}u \quad ; \quad y = [C_1, C_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [C_1x_1 + C_2\{-A_{22}^{-1}(A_{21}x_1 + B_2u)\}$$

$$= [C_1 - C_2A_{22}^{-1}A_{21}]x_1 - C_2B_2u$$

$$= \tilde{C}x_1 + Du$$

$$T_{nm}(s) = \tilde{C}(s1_n - \tilde{A})^{-1}\tilde{B} + D; \quad D = T_{nm}(\infty)$$

But T_{ran} in state eqs requires more added terms for zeroes

Companion matrix

$$v_o/v_i(s) = \frac{1}{s^3 + d_2 s^2 + d_1 s + d_0}, \quad s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -d_0 & -d_1 & -d_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_i$$

$$v_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$s^3 v_o + d_2 \underbrace{s^2 v_o}_{x_3} + d_1 \underbrace{s v_o}_{x_2} + d_0 \underbrace{v_o}_{= x_1} = v_i$$

$$s x_3 = s^3 v_o = -d_2 s^2 v_o - d_1 s v_o - d_0 \underbrace{v_o}_{= x_1} + v_i$$

But if have a numerator $T_{ran}(s) = \dfrac{n_2 s^2 + n_1 s + n_0}{s^3 + d_2 s^2 + d_1 s + d_0} + d = \dfrac{v_o}{v_i}$

$$= \frac{(N(s))x}{(D(s))x} + d$$

$$(s^3 + d_2 s^2 + d_1 s + d_0)x = v_i$$

$$(n_2 s^2 + n_1 s + n_0)x = v_o$$

$$x = x_1, \quad sx = x_2, \quad s^2 x = x_3$$

$$\underbrace{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -d_0 & -d_1 & -d_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_i \qquad v_o = \begin{bmatrix} n_0, & n_1 & n_2 \end{bmatrix} x + d v_i}_{\text{state variable equations for}}$$